Communication Systems

Lecture # 09

Amplitude Modulation

- There are several different ways of amplitude modulating the carrier signal by m(t); each results in different spectral characteristics for the transmitted signals.
  - Double Sideband- Suppressed Carrier AM
  - Conventional Double Sideband AM
  - Single Sideband AM
  - Vestigial Sideband AM

Frequency Conversion

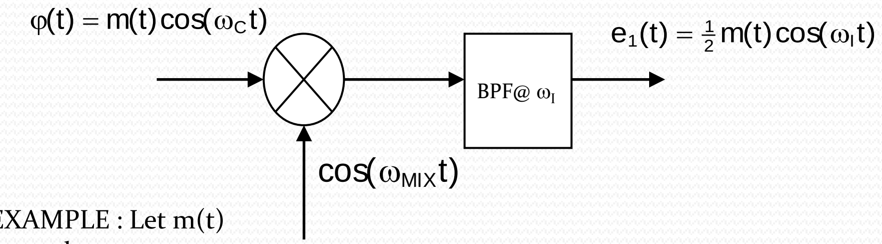
- Frequency Mixer or converter
- Used to change the frequency of a modulated signal $m(t)\cos\omega_c t$ from ω_c to some other frequency ω_I
- This can be done by multiplying $m(t)\cos\omega_c t$ by $2\cos\omega_{mix} t$ where $\omega_{mix} = \omega_c + \omega_I$ or $\omega_{mix} = \omega_c - \omega_I$
- A bandpass filter at the output, tuned to ω_I will pass the term $m(t)\cos\omega_I t$ and suppress the other terms

Frequency Conversion

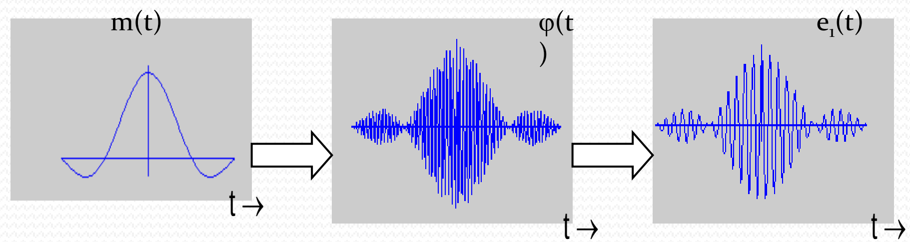
- The operation of frequency mixing (also known as heterodyning) is identical to the operation of modulation
- When we select the local carrier frequency $\omega_{mix} = \omega_c + \omega_I$, the operation is called up-conversion, and when we select $\omega_{mix} = \omega_c - \omega_I$, the operation is down conversion

Frequency Conversion

- To change the carrier frequency ωc of a modulated signal to an intermediate frequency ωI we use an oscillator to generate a sinusoid of frequency ωMIX such that



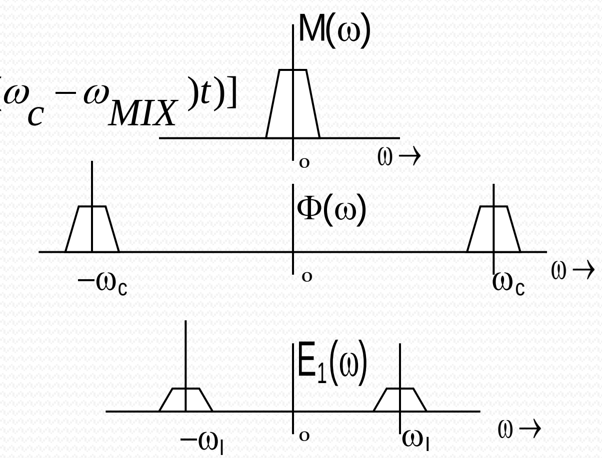
EXAMPLE : Let m(t) be as shown.



$$\omega_I = \omega_c - \omega_{MIX}.$$

Then $$m(t)\cos(\omega_c t)\cos(\omega_{MIX} t) = \frac{1}{2} m(t)[\cos((\omega_c + \omega_{MIX})t) + \cos((\omega_c - \omega_{MIX})t)]$$

$$= \frac{1}{2} m(t)[\cos((2\omega_c + \omega_I)t) + \cos((\omega_I)t)]$$



SPECTRA

Conventional Double Sideband AM

Amplitude Modulation

An alternative for the synchronous detection is to transmit a carrier $A\cos\omega_c t$ along with the modulated signal $m(t)\cos\omega_c t$ so that there is no need to generate a carrier at the receiver

In this case the transmitter needs to transmit much larger power, which makes it expensive

The transmitted signal is given by:

$$\varphi_{AM}(t) = A\cos\omega_c t + m(t)\cos\omega_c t$$
$$= [A + m(t)]\cos\omega_c t$$

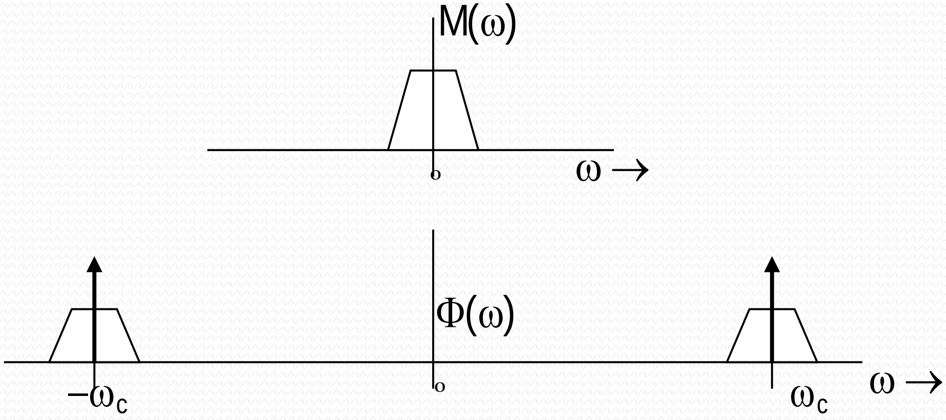
$$\varphi_{AM}(t) \leftrightarrow \frac{1}{2}[M(\omega+\omega_c)+M(\omega-\omega_c)]+\pi A[\delta(\omega+\omega_c)+\delta(\omega-\omega_c)]$$

Amplitude Modulation

- Why DSB-SC not working: do not know the carrier frequency in receiver.
- The last impulse functions indicate that the carrier is not suppressed in this case. For some M() shown, the modulated signal spectrum is as shown.

$$\varphi_{AM}(t) = [A + m(t)]\cos(\omega_c t)$$

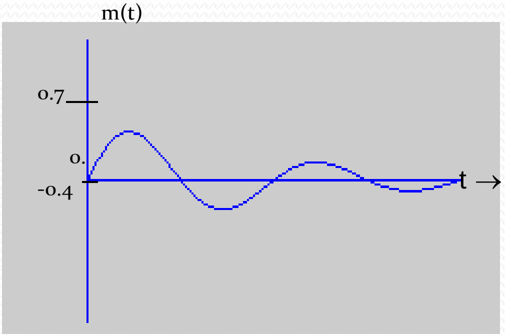
$$\Phi(\omega) = \tfrac{1}{2}\left[M(\omega-\omega_c) + M(\omega-\omega_c)\right] + A\pi\left[\delta(\omega-\omega_c) + \delta(\omega+\omega_c)\right]$$

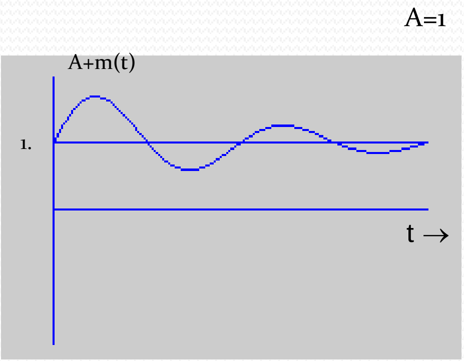


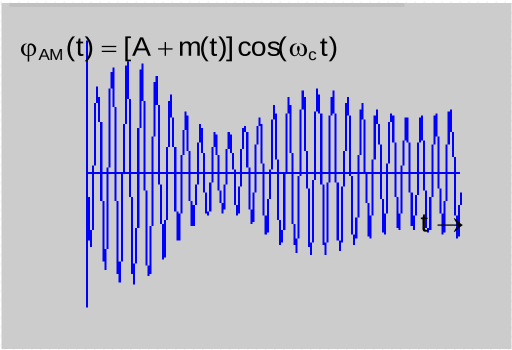
- With this type of AM the demodulation can be performed with/without a local oscillator synchronized with the transmitter.

AM Example

- m(t) has a minimum value of about -0.4. Adding a dc offset of A=1 results in A+m(t) being always positive. Therefore the positive envelope of is just A+m(t). An envelope detector can be used to retrieve this.

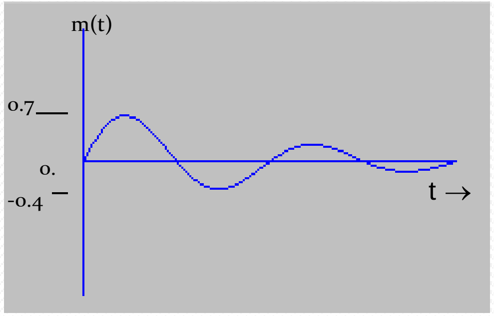


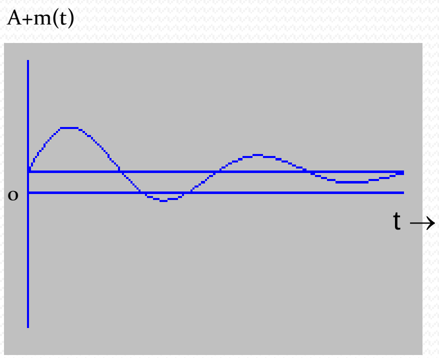


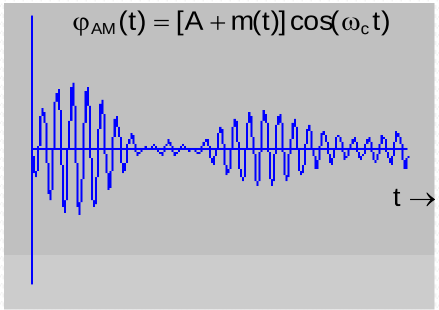


AM Example (contd.)

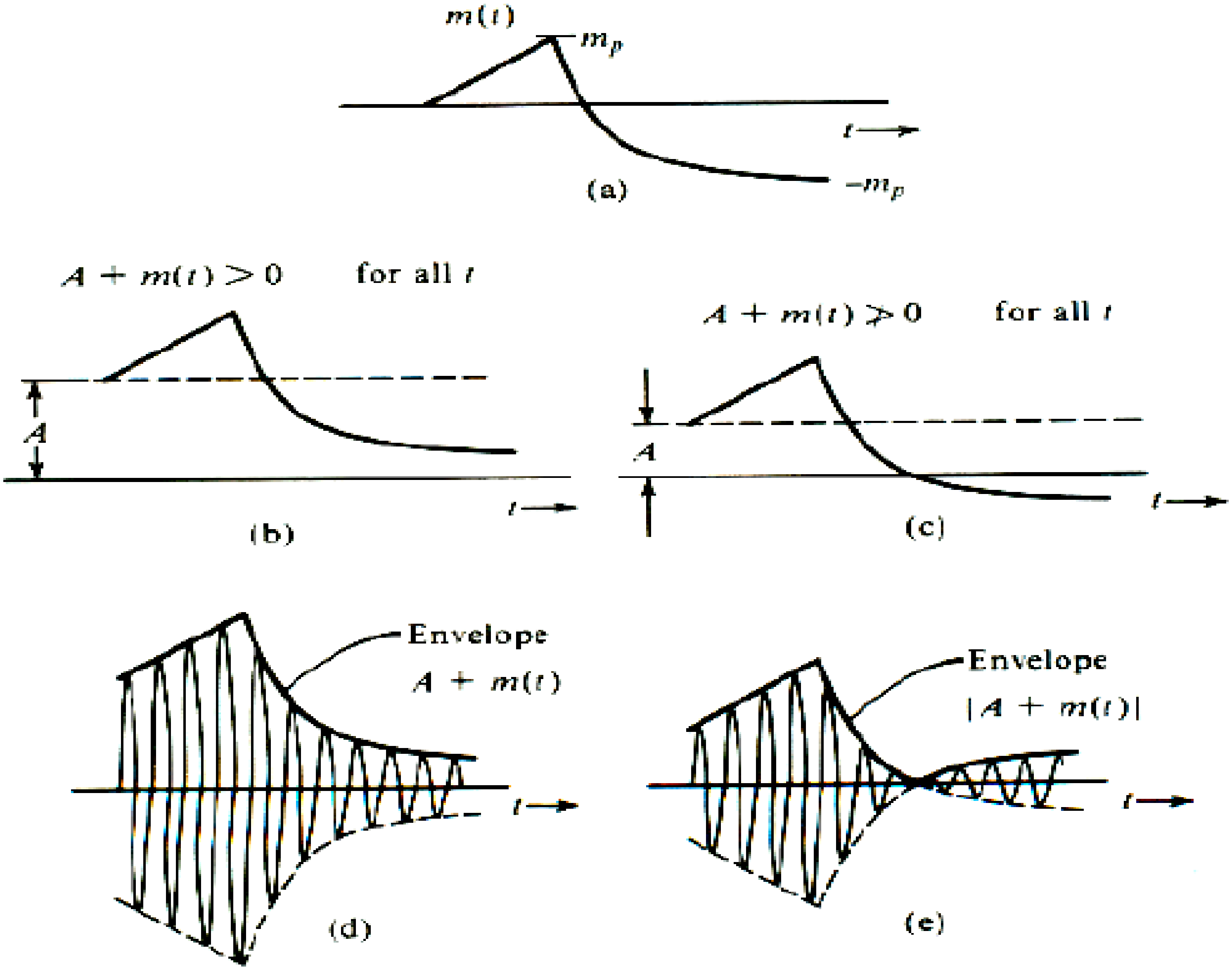
- The choice of dc offset should be such that A+m(t) should always be positive. Otherwise envelope detector cannot be used, but coherent still ok
- For example, the minimum value of m(t) = -0.4 . Therefore A > |min(m(t))| for successful envelope detection. What if A< |m(t) |.
- In the previous example let A=0.3.







Amplitude Modulation



Amplitude Modulation

- Condition for envelop detection

$$A + m(t) \geq 0 \quad \text{for all } t$$

Minimum carrier amplitude required for viability of envelop detection is m_p

- Modulation index

$$\mu = \frac{m_p}{A}$$

$$0 \leq \mu \leq 1$$

Note that the synchronous detection can be used for any value of μ. The envelop detector, which is considerably simpler and

Amplitude Modulation

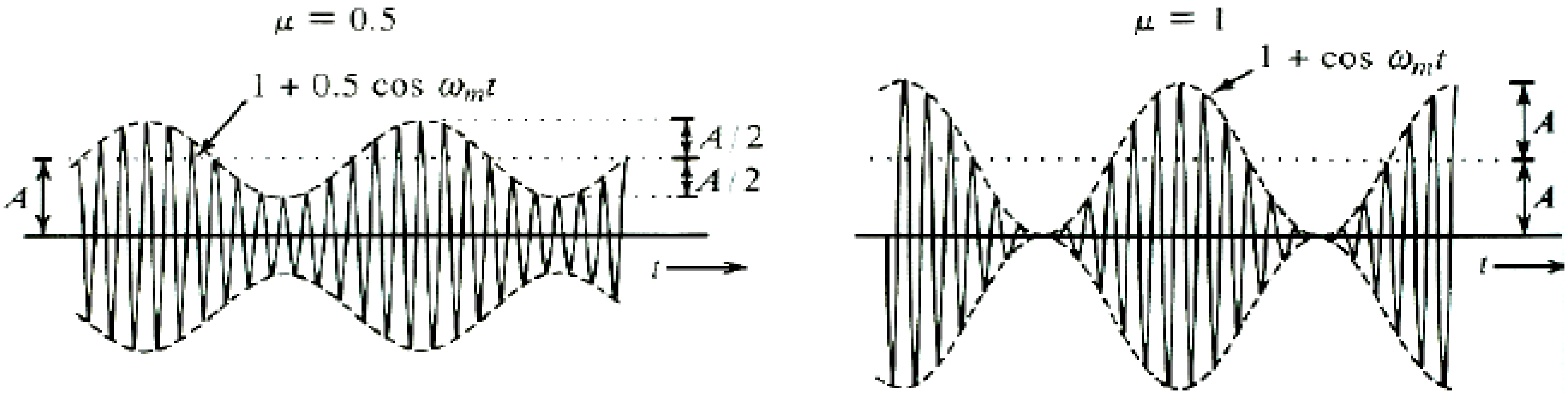
less expensive than the synchronous detector

can be used only for $\mu \leq 1$

Example

Sketch $\varphi_{AM}(t)$ for modulation indices of $\mu = 0.5$ and $\mu = 1$ when $m(t) = B\cos\omega_m t$

TONE MODULATION



Modulation Index

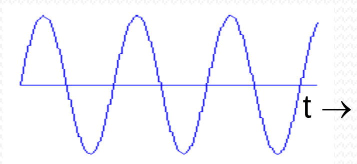
$A \geq m_p$ $\underline{A \text{ is the carrier amplitude}}.$

MODULATION INDEX : $\mu = \dfrac{m_p}{A}$

Then we see that for $A \geq m_p$, $\underline{\underline{0 \leq \mu \leq 1}}$

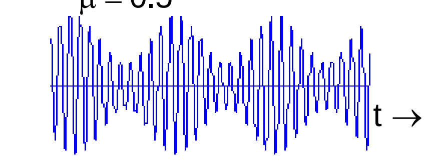
When $\mu > 1$ (or $A < m_p$) the signal is overmodulated, and envelope detection can not be used.
(However, we can still use synchronous demodulation).

m(t)

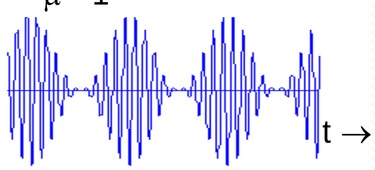


$m_p = 2; \quad \mu = \dfrac{m_p}{A} = \dfrac{2}{A}.$ i) $\mu = 0.5 \quad A = 4$ ii) $\mu = 1 \quad A = 2$
For dc offset of 1 $\mu = 2$.

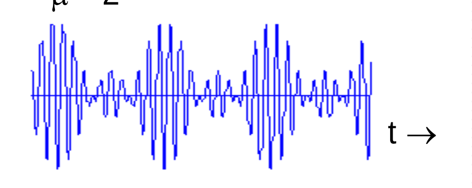
$\mu = 0.5$



$\mu = 1$

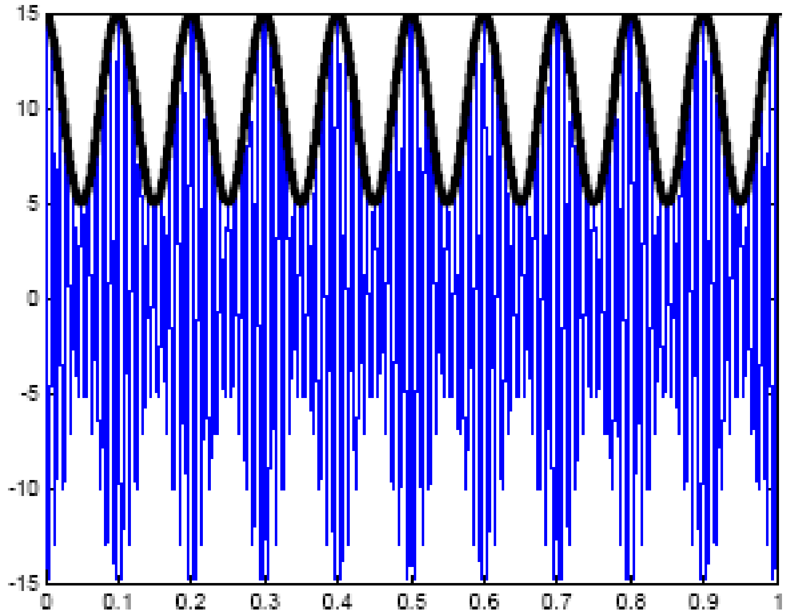


$\mu = 2$

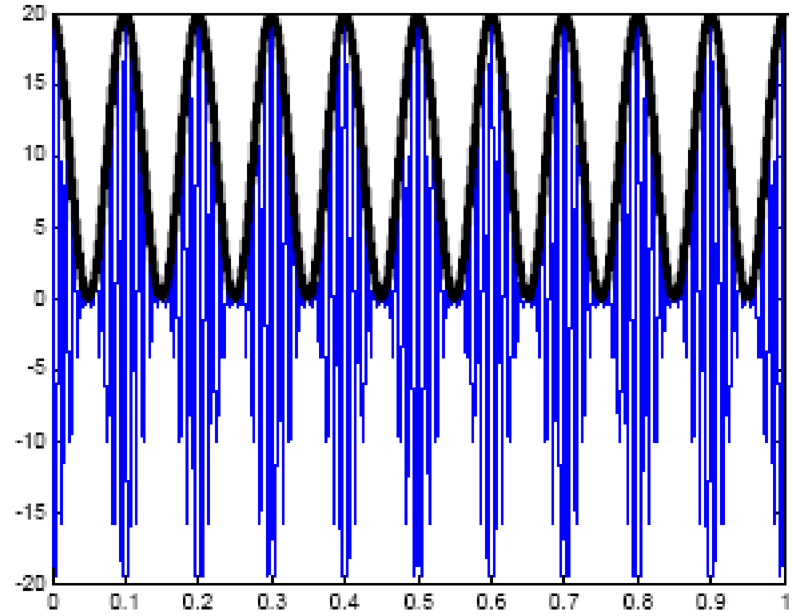


AM Example

Mu=0.5



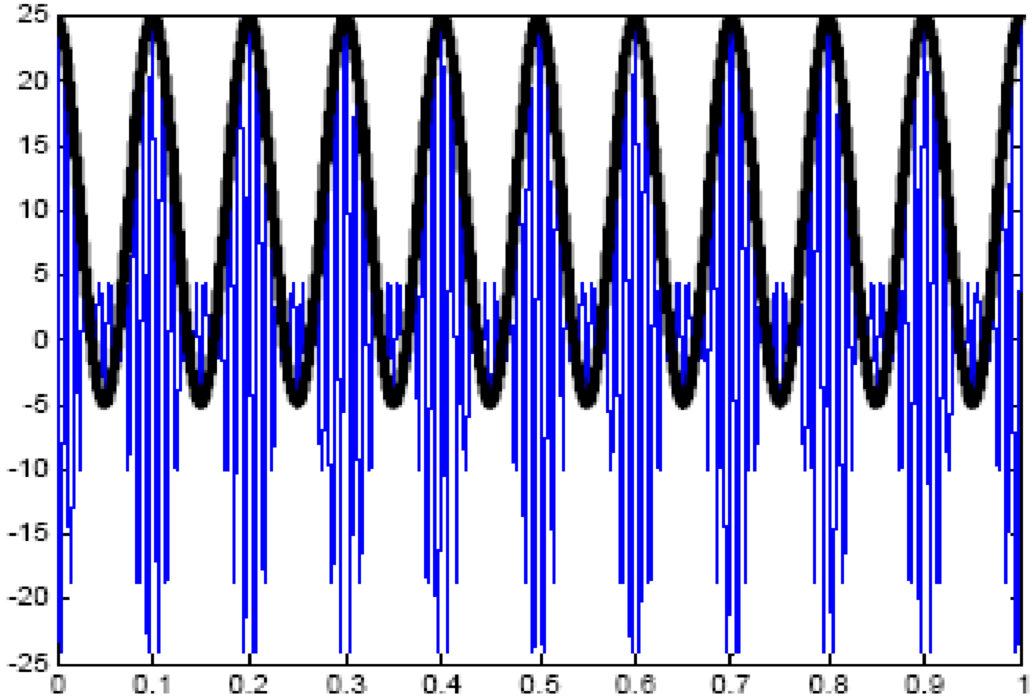
Mu=1



Envelope Detection can be used to recover the signal

Over-modulation Case

Mu=1.5



Only Synchronous detection can recover the signal

Amplitude Modulation

Sideband and Carrier Power

In AM the carrier term does not carry any information, and hence the carrier power is wasted

$$\varphi_{AM}(t) = \underbrace{A \cos \omega_c t}_{\text{carrier}} + \underbrace{m(t) \cos \omega_c t}_{\text{sidebands}}$$

$$P_c = \frac{A^2}{2} \qquad \text{and} \qquad P_s = \frac{1}{2}\widetilde{m^2(t)}$$

$$\eta = \frac{\text{useful power}}{\text{total power}} = \frac{P_s}{P_c + P_s} = \frac{\widetilde{m^2(t)}}{A^2 + \widetilde{m^2(t)}} 100\%$$

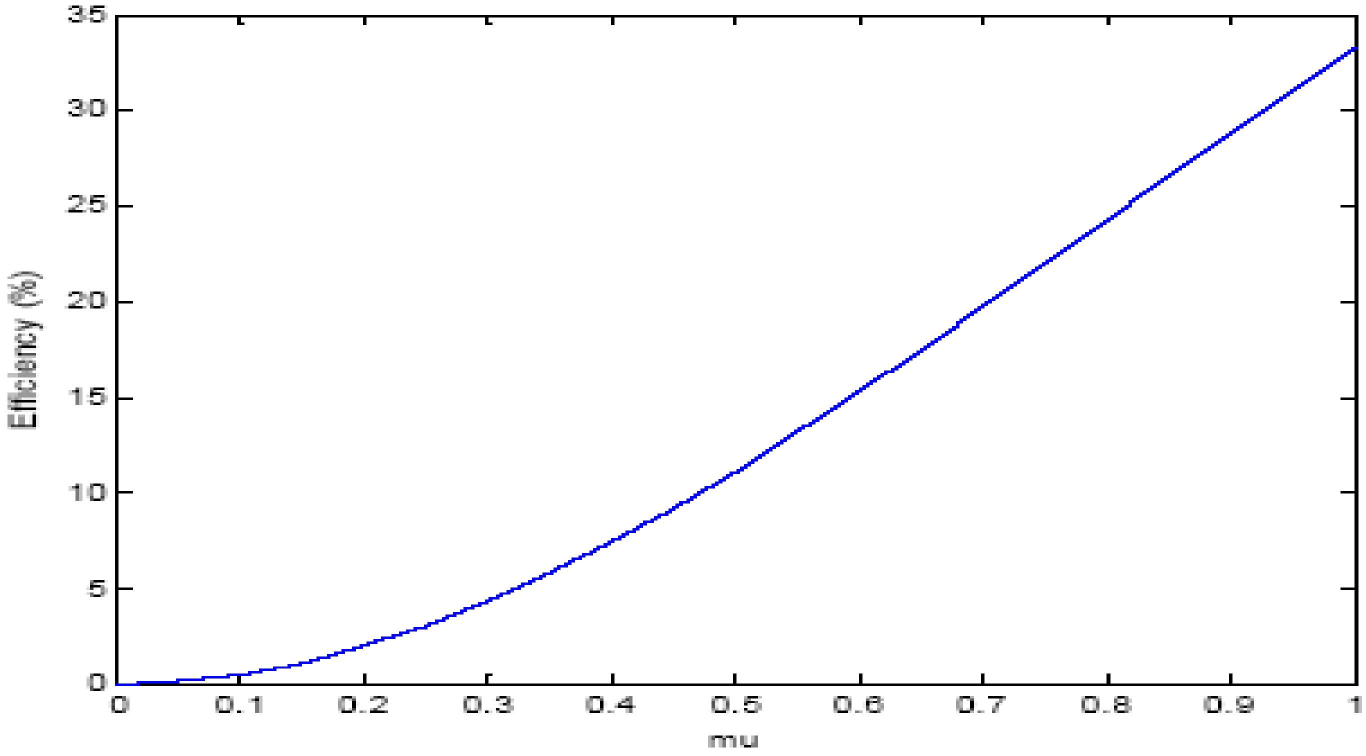
Tone Modulation

$$m(t) = \mu A \cos \omega_m t \qquad \text{and} \qquad \widetilde{m^2(t)} = \frac{(\mu A)^2}{2}$$

$$\eta = \frac{\mu^2}{2 + \mu^2} 100\%$$

Example

Tone Modulation



Maximum efficiency of 33% occurs at mu=1
Under best conditions

Generation of AM signal

- AM genarator

AM Decoder

- Rectifier Detector: synchronous
- Envelope Detector: asynchronous

Envelop Detector

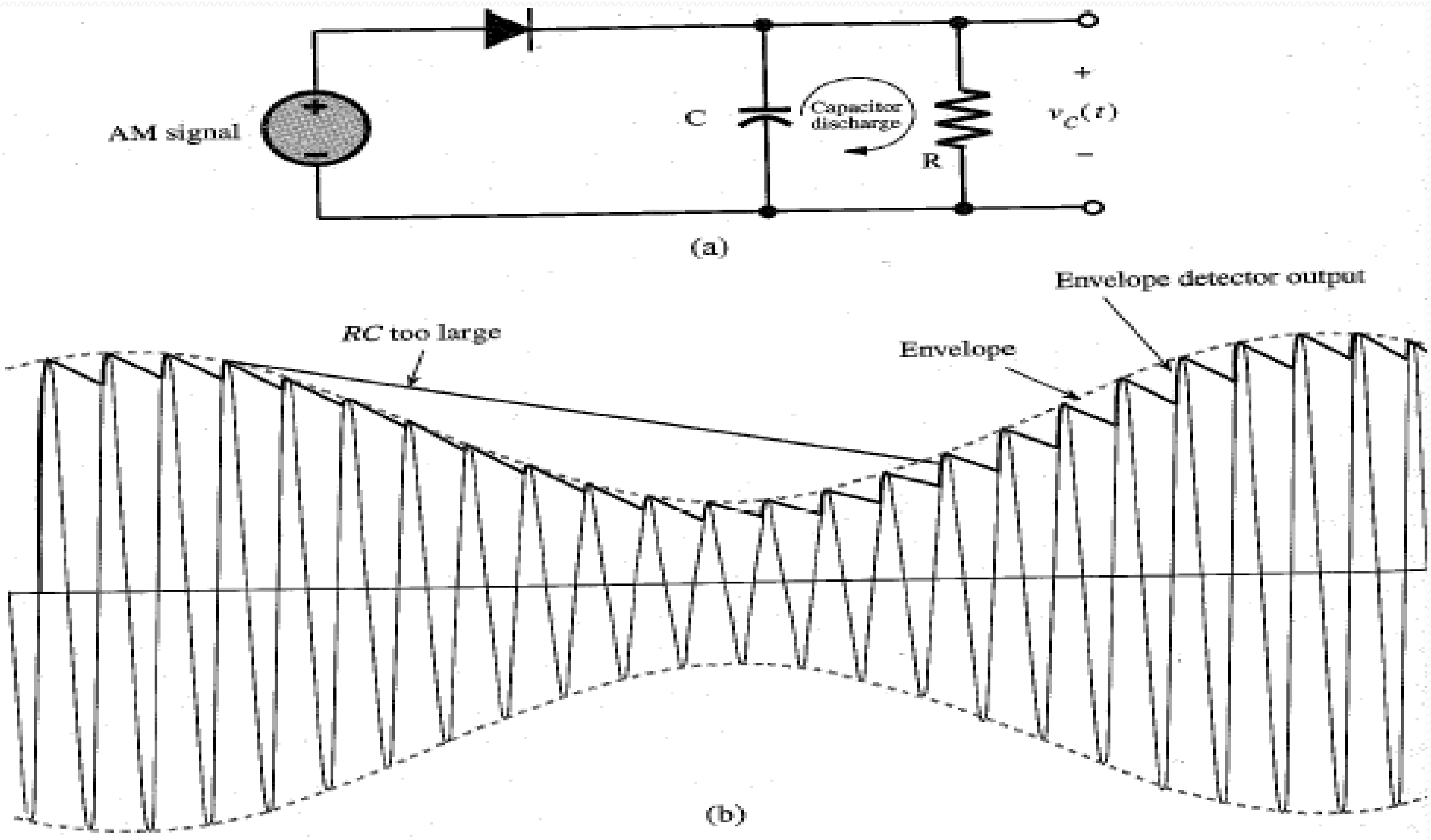
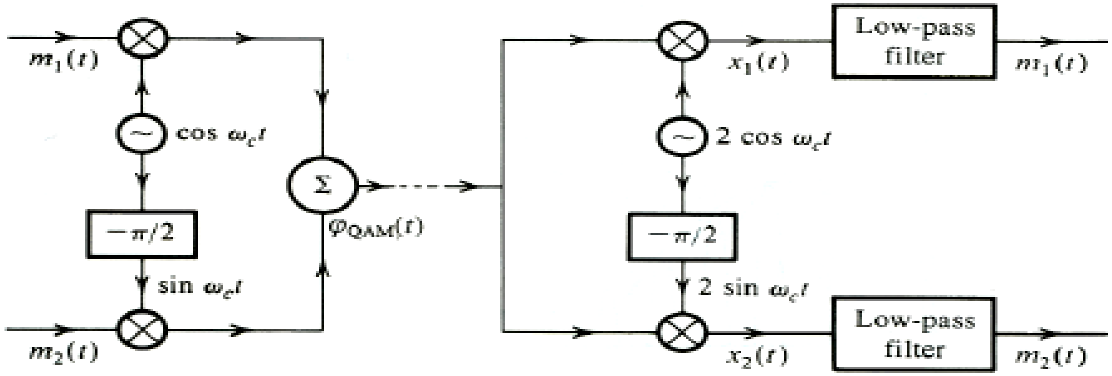


Figure 4.12 Envelope detector for AM.

QAM

- Quadrature Amplitude Modulation (QAM) The DSB signals occupy twice the bandwidth required for the baseband
- Transmitting two baseband signals using carriers of the same frequency but in phase quadrature



$$\varphi_{QAM}(t) = m_1(t)\cos \omega_c t + m_2(t)\sin \omega_c t$$

QAM (contd.)

- Both modulated signals occupy the same band and can be separated at the receiver by using synchronous detection using two local carriers in phase quadrature
- The upper branch is also known as in-phase (I) channel and the lower branch is the quadrature (Q) channel
- A slight error in the phase or frequency of the local carriers at the demodulator will not only distort the signal but will also lead to interference between two channels

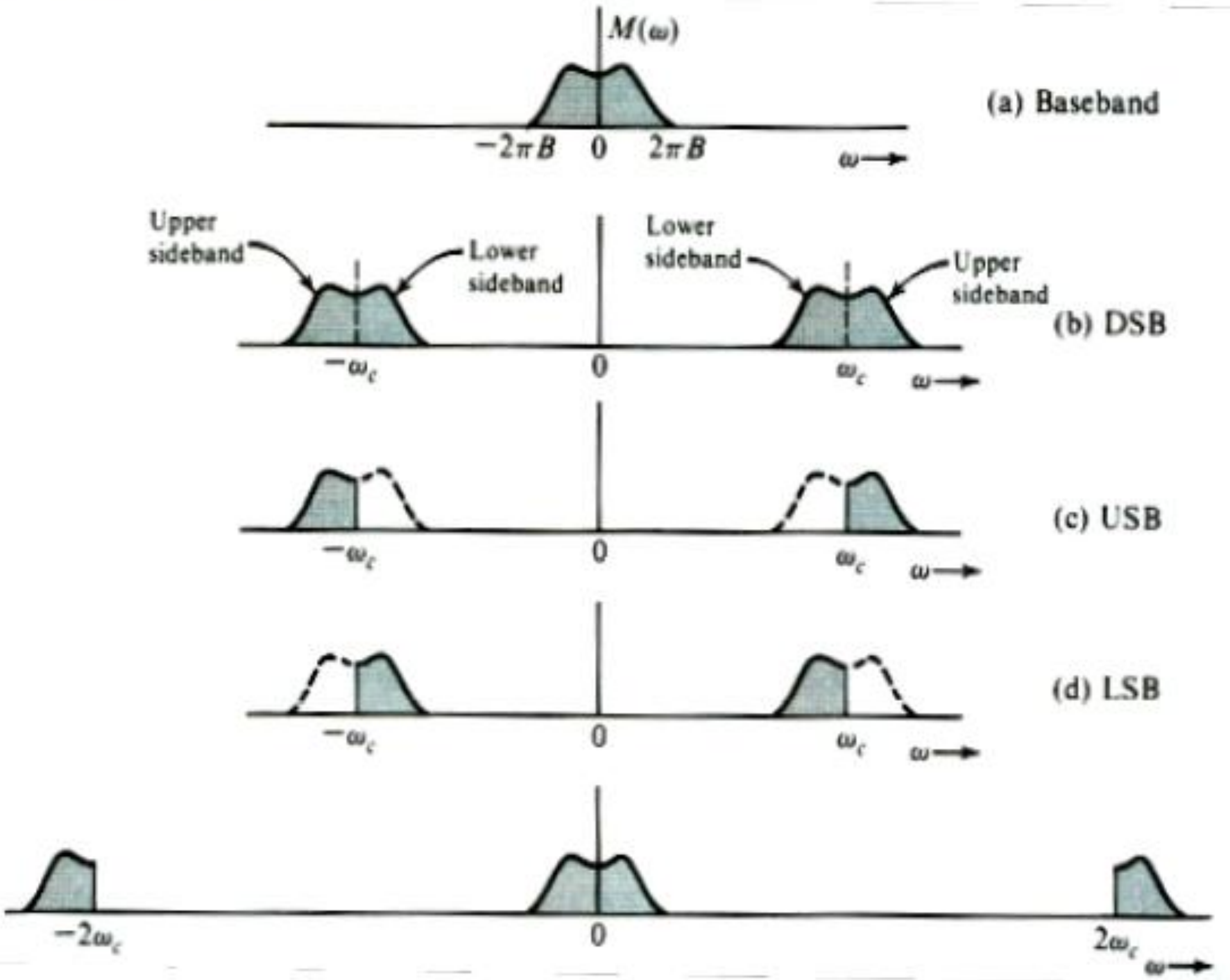
QAM

- AM signal BANDWIDTH : AM signal bandwidth is twice the bandwidth of the modulating signal. A 5kHz signal requires 10kHz bandwidth for AM transmission. If the carrier frequency is 1000 kHz, the AM signal spectrum is in the frequency range of 995kHz to 1005 kHz.
- QUADRARTURE AMPLITUDE MODULATION is a scheme that allows two signals to be transmitted over the same frequency range.
- Coherent in frequencyand phase.

Amplitude Modulation: Single Sideband

- Amplitude Modulation Single Side Band Also known as SSB-SC
- A scheme in which only one sideband (USB or LSB) is transmitted
- Requires only one half the bandwidth of the DSB signal
- Demodulation of SSB signals is identical to that of DSB-SC signals

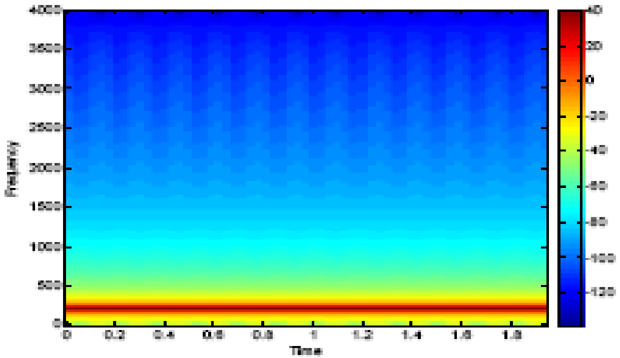
SSB Spectra



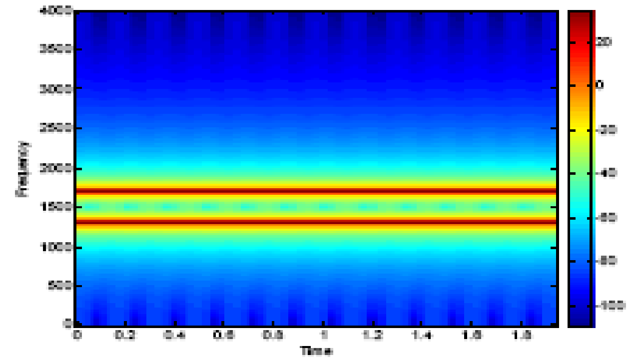
M(ω)
(a) Baseband
−2πB
0
2πB
ω→
Upper sideband
Lower sideband
Lower sideband
Upper sideband
(b) DSB
−ω_c
0
ω_c
ω→
(c) USB
−ω_c
0
ω_c
ω→
(d) LSB
−ω_c
0
ω_c
ω→
−2ω_c
0
2ω_c
ω→

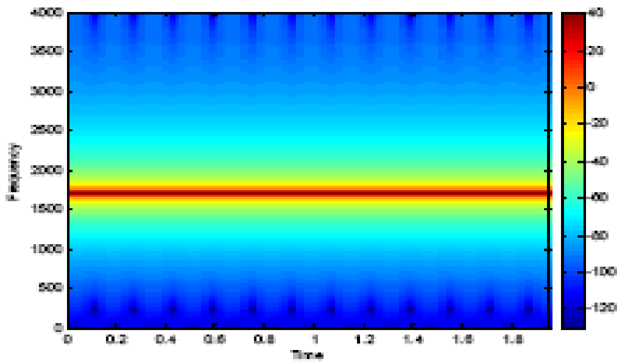
Motivation
Tone Modulation: Simple Sinusoid Case

100 Hz

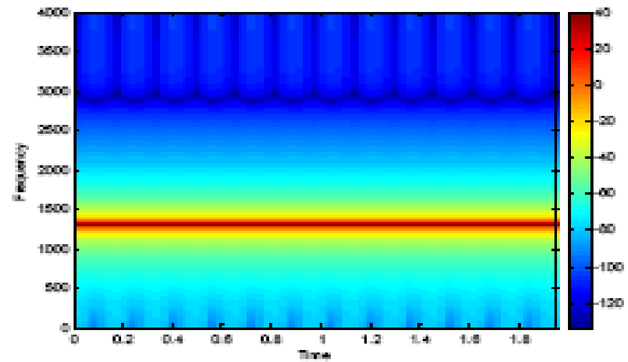


DSB-SC, 1500 Hz Carrier





SSB-USB



SSB-LSB

Time Domain Representation of SSB Signals

1	$2\pi\delta(\omega)$
$u(t)$	$\pi\delta(\omega) + \dfrac{1}{j\omega}$
$\text{sgn}\, t$	$\dfrac{2}{j\omega}$

$$M_+(\omega) = M(\omega)U(\omega) = M(\omega)\left[\frac{1+\text{sgn}(\omega)}{2}\right] = \tfrac{1}{2}\left[M(\omega) + M(\omega)\text{sgn}(\omega)\right] \quad \Leftrightarrow \quad m_+(t) = \tfrac{1}{2}\text{m(t)} + \tfrac{1}{2}\mathcal{F}^{-1}\{M(\omega)\} * \mathcal{F}^{-1}\{\text{sgn}(\omega)\}$$

$$\mathcal{F}^{-1}\{\text{sgn}(\omega)\} = -\frac{1}{j\pi t} = \frac{j}{\pi t} \qquad \therefore \quad m_+(t) = \tfrac{1}{2}\left(\text{m(t)} + jm(t) * \frac{1}{\pi t}\right) = \tfrac{1}{2}\left(\text{m(t)} + jm_h(t)\right)$$

$$\text{where } m_h(t) = m(t) * \frac{1}{\pi t} = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{m(\lambda)}{t-\lambda} d\lambda$$

$m_h(t)$ is called the Hilbert transform of m(t).

Similarly, we can show that $m_-(t) = \tfrac{1}{2}\left(\text{m(t)} - jm(t) * \frac{1}{\pi t}\right) = \tfrac{1}{2}\left(\text{m(t)} - jm_h(t)\right)$

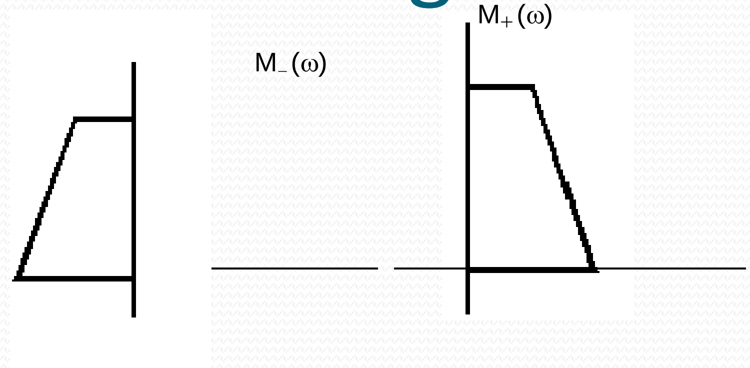
$$\mathbf{M_H(\omega)} = -j\,\mathbf{M(\omega)\ sgn(\omega)}$$

Time Domain Representation of SSB Signals

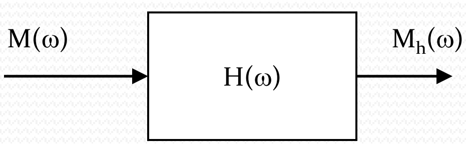
$$M_H(\omega) = -jM(\omega)\,\mathrm{sgn}(\omega)$$

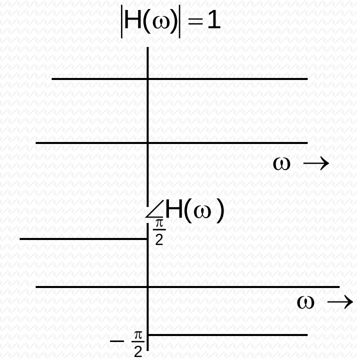
$$H(\omega) = \frac{M_H(\omega)}{M(\omega)} = -j\,\mathrm{sgn}(\omega)$$

$$= \begin{cases} -j & \text{for } \omega > 0 \\ j & \text{for } \omega < 0 \end{cases}$$



Transfer function of a Hilbert transformer



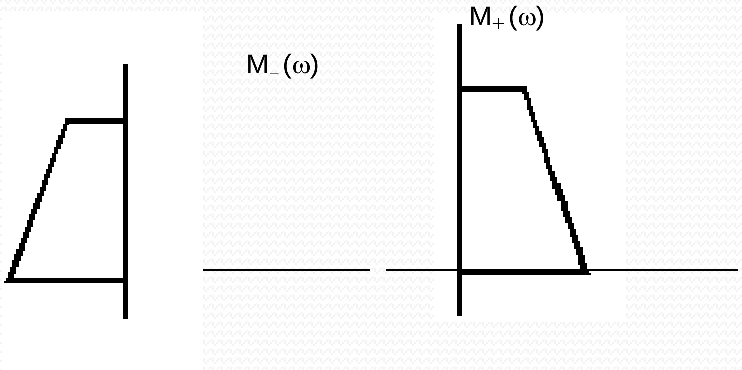


Time Domain Representation of SSB Signals

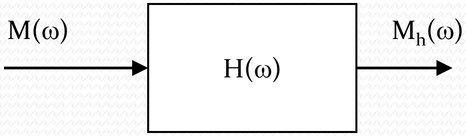
$$M_H(\omega) = -jM(\omega)\,\text{sgn}(\omega)$$

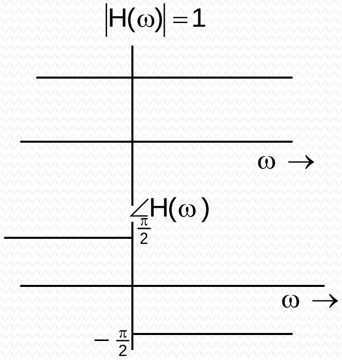
$$H(\omega) = \frac{M_H(\omega)}{M(\omega)} = -\,j\text{sgn}(\omega)$$

$$= \begin{cases} -j & \text{for } \omega > 0 \\ j & \text{for } \omega < 0 \end{cases}$$



Transfer function of a Hilbert transformer





Time Domain Representation of SSB Signals

- SSB signal can be expressed in terms of m(t) and its Hilbert transform

$$\Phi_{SSB-USB}(\omega) = \left(M_+(\omega-\omega_c) + M_-(\omega+\omega_c)\right)$$

$$M_+(\omega-\omega_c) = \mathcal{F}^{-1}\{m_+(t)e^{j\omega_c t}\} = \mathcal{F}^{-1}\{\tfrac{1}{2}(m(t)+jm_h(t))e^{j\omega_c t}\}$$

$$M_-(\omega+\omega_c) = \mathcal{F}^{-1}\{m_-(t)e^{-j\omega_c t}\} = \mathcal{F}^{-1}\{\tfrac{1}{2}(m(t)-jm_h(t))e^{-j\omega_c t}$$

$$\therefore M_+(\omega-\omega_c) + M_-(\omega+\omega_c) = \mathcal{F}^{-1}\{\tfrac{1}{2}(m(t)+jm_h(t))e^{j\omega_c t} + \tfrac{1}{2}(m(t)-jm_h(t))e^{-j\omega_c t}\}$$

$$= \mathcal{F}^{-1}\left\{\tfrac{1}{2}m(t)(e^{j\omega_c t} + e^{j\omega_c t}) + j\tfrac{1}{2}m_h(t)(e^{j\omega_c t} - e^{-j\omega_c t})\right\}$$

$$= \mathcal{F}^{-1}\{m(t)\cos(\omega_c t) - m_h(t)\sin(\omega_c t)\}$$

$$\therefore \quad \Phi_{SSB-USB}(\omega) = \mathcal{F}^{-1}\{m(t)\cos(\omega_c t) - m_h(t)\sin(\omega_c t)\}$$

Similarly we can show that $\Phi_{SSB-LSB}(\omega) = \mathcal{F}^{-1}\{m(t)\cos(\omega_c t) + m_h(t)\sin(\omega_c t)\}$;

In general, $\Phi_{SSB}(\omega) = \mathcal{F}^{-1}\{m(t)\cos(\omega_c t) \mp m_h(t)\sin(\omega_c t)\}$ (- for USB, + for LSB)

Time Domain Representation of SSB Signals

$$\varphi_{USB}(t) = m(t)\cos\omega_c t - m_h(t)\sin\omega_c t$$

$$\varphi_{LSB}(t) = m(t)\cos\omega_c t + m_h(t)\sin\omega_c t$$

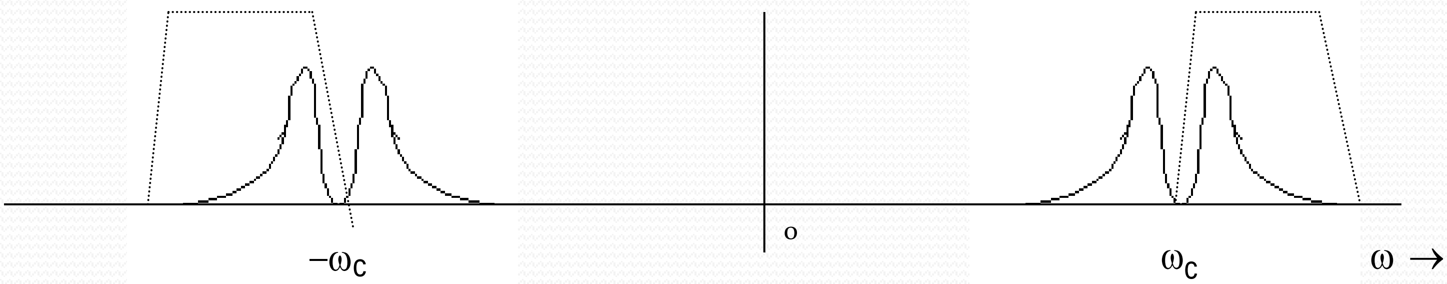
$$\varphi_{SSB}(t) = m(t)\cos\omega_c t \mp m_h(t)\sin\omega_c t$$

Generation of SSB

- Selective filtering Method
  - A DSB-SC signal is passed through a sharp cuttoff filter to eliminate the undesired sideband
  - The filter should pass all components above w_c untenanted and completely suppress all components below w_c
  - Such an operation require an ideal filter, which is unrealizable
  - It can be realized closely if there is some separation between the passband and stopband e.g. Speech Signal (For speech frequencies below 300 Hz can be removed without effecting the quality)

SSB Generator

- Selective Filtering using filters with sharp cutoff characteristics. Sharp cutoff filters are difficult to design. The audio signal spectrum has no dc component, therefore , the spectrum of the modulated audio signal has a null around the carrier frequency. This means a less than perfect filter can do a reasonably good job of filtering the DSB to produce SSB signals.
- Baseband signal must be bandpass
- Filter design challenges
- No low frequency components



SSB Demodulation

Synchronous, SSB-SC demodulation

$$\varphi_{SSB}(t)\cos(\omega_c t) = [m(t)\cos(\omega_c t) \mp j m_h(t)\sin(\omega_c t)]\cos(n(\omega_c t) = \frac{1}{2}[m(t)(1+\cos(\omega_c t)) \mp j m_h(t)\sin(2\omega_c t)]$$

A lowpass filter can be used to get $\frac{1}{2}m(t)$.

SSB+C, envelop detection

$$\varphi_{SSB+C}(t) = A\cos(\omega_c t) + [m(t)\cos(\omega_c t) \mp m_h(t)\sin(\omega_c t)]$$

An envelope detector can be used to demodulate such SSB signals.

What is the envelope of $\varphi_{SSB+C}(t) = (A + m(t))\cos(\omega_c t)) + m_h(t)\sin(\omega_c t) = E(t)\cos(\omega_c t + \theta)$?

{Recall $A\cos(\alpha) + B\sin(\alpha) = (A^2 + B^2)^{\frac{1}{2}}\cos(\alpha + \theta),\ \theta = -\tan^{-1}(\frac{B}{A}))$

$$E(t) = ((A + m(t))^2 + m_h^2(t))^{\frac{1}{2}} = ((A^2 + m^2(t)) + m_h^2(t) + 2Am(t))^{\frac{1}{2}}$$

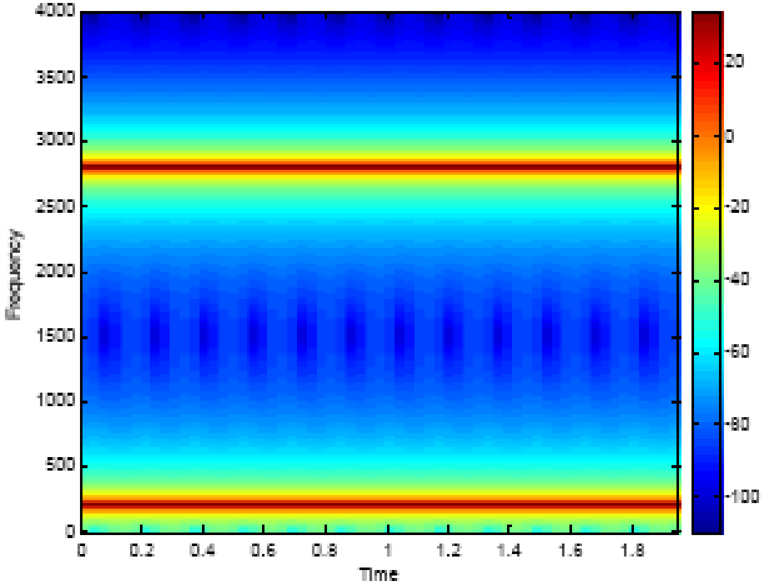
$$= A\left(1 + \frac{m^2(t)}{A^2} + \frac{m_h{}^2(t)}{A^2} + \frac{2m(t)}{A}\right)$$

$$\approx A + m(t) \qquad \text{for } A \gg |m(t)|,\ A \gg |m_h(t)|.$$

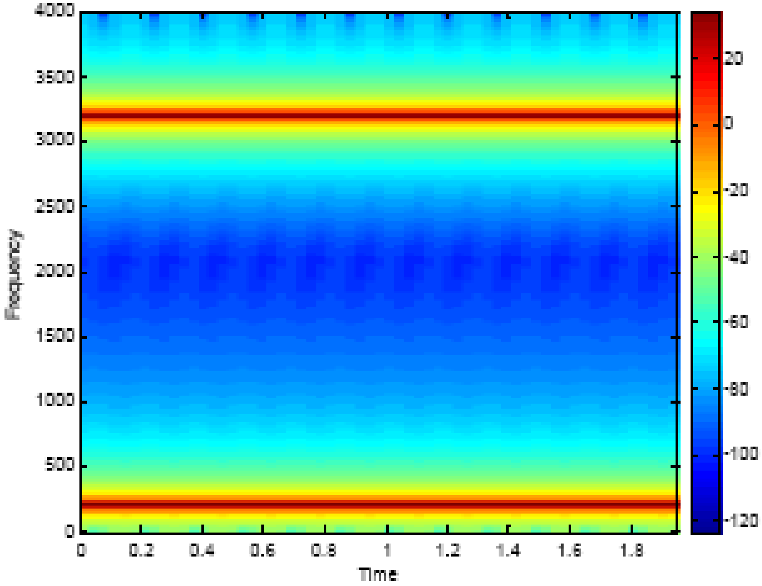
The efficiency of this scheme is very low since A has to be large.

Demodulation of SSB

Synchronous Demodulation can be used for SSB-SC signals



SSB-LSB



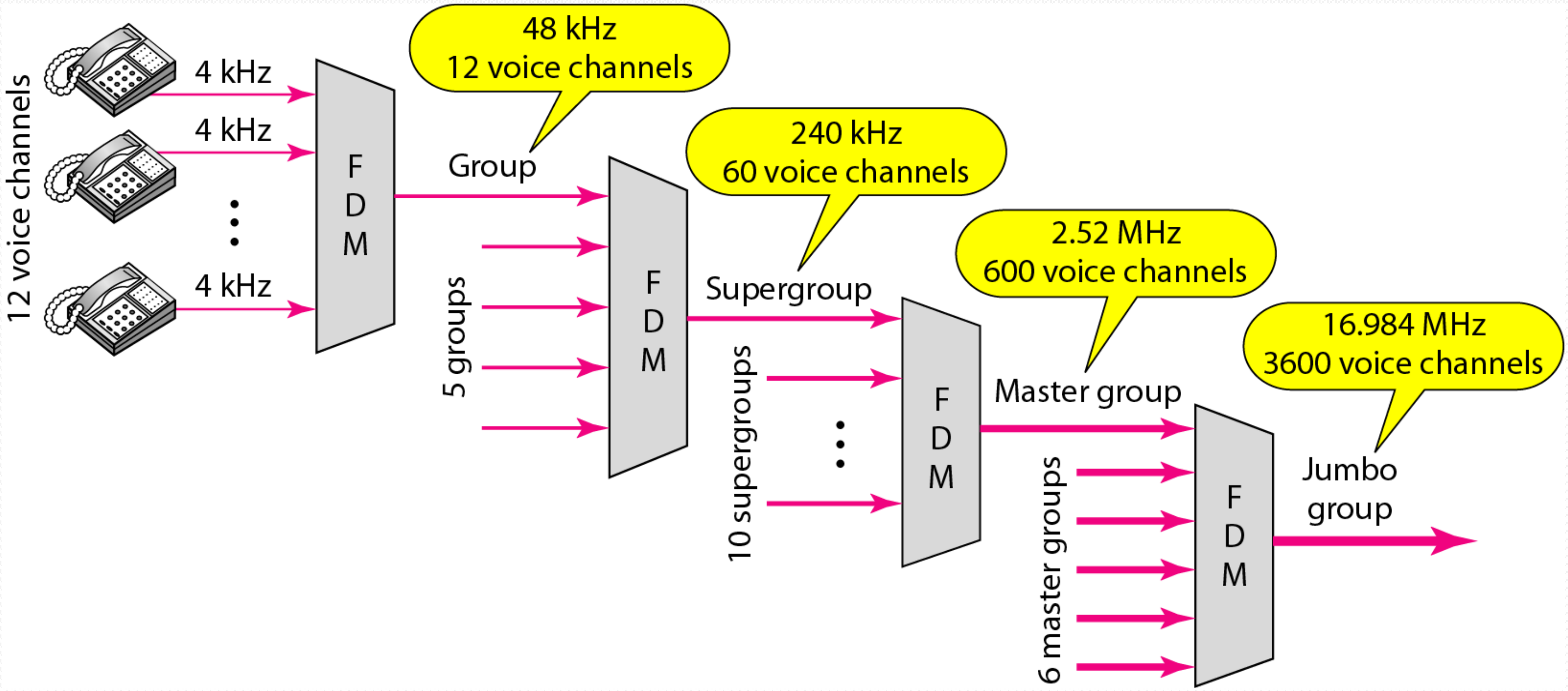
SSB-USB

SSB vs. AM

- Since the carrier is not transmitted, there is a reduction by 67% of the transmitted power (-4.7dBm). --In AM @100% modulation: 2/3 of the power is comprised of the carrier; with the remaining (1/3) power in both sidebands.
- Because in SSB, only one sideband is transmitted, there is a further reduction by 50% in transmitted power
- Finally, because only one sideband is received, the receiver's needed bandwidth is reduced by one half--thus effectively reducing the required power by the transmitter another 50%
- (-4.7dBm (+) -3dBm (+) -3dBm = -10.7dBm).
- Relative expensive receiver

Telephone Channel Multiplexing

Almost all long-haul telephone channels were multiplexed by FDM using SSB signals.



Questions?